The Dark Side of the Solar Neutrino Parameter Space*

André de Gouvêa
CERN - Theory Division, CH-1211 Geneva 23, Switzerland

Alexander Friedland and Hitoshi Murayama
Department of Physics, University of California, Berkeley, CA 94720, USA
Theory Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
(February 1, 2008)

Results of neutrino oscillation experiments have always been presented on the $(\sin^2 2\theta, \Delta m^2)$ parameter space for the case of two-flavor oscillations. We point out, however, that this parameterization misses the half of the parameter space $\frac{\pi}{4} < \theta \leq \frac{\pi}{2}$ ("the dark side"), which is physically inequivalent to the region $0 \leq \theta \leq \frac{\pi}{4}$ ("the light side") in the presence of matter effects. The MSW solutions to the solar neutrino problem can extend to the dark side, especially if we take the conservative attitude to allow higher confidence levels, ignore some of the experimental results in the fits, or relax theoretical predictions. Furthermore, even the so-called "vacuum oscillation" solution distinguishes the dark and the light sides. We urge experimental collaborations to present their results on the entire parameter space.

arXiv:hep-ph/0002064v2 20 Feb 2000

In the Standard Model of particle physics, neutrinos are strictly massless. Recently, however, the Super-Kamiokande collaboration studied atmospheric neutrinos and reported a strong evidence for neutrino oscillations [1], and hence a finite neutrino mass. The most likely interpretation of their data is the oscillation between ν_μ and ν_τ. This made it also natural to interpret another long-standing issue in neutrino physics, the deficit of the solar ν_e flux [2], in terms of neutrino oscillations. However, the solar neutrino deficit has not been regarded as convincing evidence for neutrino oscillations in the community. The reason is probably multifold but two main objections are the following. (1) Neutrino experiments are so difficult that it is possible that some of the data are not entirely correct. (2) The physics of the Sun is so complex that the neutrino flux calculations in the Standard Solar Model (SSM) may have underestimated the theoretical uncertainties.

To resolve this situation, a new generation of solar neutrino experiments, such as Super-Kamiokande, SNO, Borexino, GNO, KamLAND, etc, is looking for an evidence for solar neutrino oscillations without relying on the SSM in well-understood experimental environments. They aim not only at establishing oscillations but also at overdetermining the solution in the next few years. Such data will eventually supersede data from the past experiments. It is therefore important to analyze the future data without too much prejudice based on the past data.

In this letter, we point out that the study of neutrino oscillations on the $(\Delta m^2, \sin^2 2\theta)$ parameter space done traditionally is incomplete, since it covers only the range $0 \leq \theta \leq \frac{\pi}{4}$ ("the light side"). Indeed, some of the solutions to the solar neutrino puzzle extend to the other half of the parameter space $\frac{\pi}{4} < \theta \leq \frac{\pi}{2}$, which we call "the dark side," and hence it is phenomenologically necessary to include both halves of the parameter space. This is especially true once one employs a more conservative attitude which either allows higher confidence levels, ignores some of the experimental data (especially Homestake [3]), or relaxes the theoretical prediction on the ^{8}B flux.

Neutrino oscillations occur if neutrino mass eigenstates are different from neutrino weak eigenstates. Assuming that only two neutrino states mix, the relation between mass eigenstates (ν_1 and ν_2) and flavor eigenstates (for example ν_e and ν_μ) is simply given by

$$|\nu_1\rangle = \cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle,$$
$$|\nu_2\rangle = \sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle, \tag{1}$$

where θ is the vacuum mixing angle. The mass-squared difference is defined as $\Delta m^2 \equiv m_2^2 - m_1^2$. We are interested in the range of parameters that encompasses all physically different situations. First, observe that Eq. (1) is invariant under $\theta \to \theta+\pi$, $|\nu_e\rangle \to -|\nu_e\rangle$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$, and hence the ranges $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $[\frac{\pi}{2}, \frac{3\pi}{2}]$ are physically equivalent. Next, note that it is also invariant under $\theta \to -\theta$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$, $|\nu_2\rangle \to -|\nu_2\rangle$, hence it is sufficient to only consider $\theta \in [0, \frac{\pi}{2}]$. Finally, it can also be made invariant under $\theta \to \frac{\pi}{2} - \theta$, $|\nu_\mu\rangle \to -|\nu_\mu\rangle$ by relabeling the mass eigenstates $|\nu_1\rangle \leftrightarrow |\nu_2\rangle$, *i.e.* $\Delta m^2 \to -\Delta m^2$. Thus, we can take $(\Delta m^2 > 0)$ without loss of generality. All physically different situations are obtained by allowing $0 \leq \theta \leq \frac{\pi}{2}$.

For the case of oscillations in the vacuum, the survival probability is given by

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2}{E}L\right). \tag{2}$$

Here, Δm^2 is given in eV^2/c^4, E in GeV, and L in km. In this case the oscillation phenomenon can be parameterized by Δm^2 and $\sin^2 2\theta$, since θ and $\frac{\pi}{2} - \theta$ yield identical physics. Therefore we can restrict ourselves to $0 \leq \theta \leq \frac{\pi}{4}$, and use the parameter space $(\Delta m^2, \sin^2 2\theta)$ without any ambiguity. This is indeed an adequate pa-

rameterization for reactor antineutrino oscillation experiments, short-baseline accelerator neutrino oscillation experiments, and $\nu_\mu \leftrightarrow \nu_\tau$ atmospheric neutrino oscillation experiments.

In certain cases, however, neutrino-matter interactions can dramatically change the oscillation probability [4]. These matter effects are particularly important in explaining the solar ν_e flux deficit in terms of neutrino oscillations. In the presence of matter effects, Eq. (2) is modified to

$$P(\nu_e \to \nu_e) = P_1 \cos^2\theta + (1-P_1)\sin^2\theta$$
$$-\sqrt{P_c(1-P_c)}\cos 2\theta_M \sin 2\theta \cos\left(2.54\frac{\Delta m^2}{E}L + \delta\right), \quad (3)$$

where P_c is the hopping probability, θ_M is the mixing angle at the production point, $P_1 = P_c \sin^2\theta_M + (1-P_c)\cos^2\theta_M$, and δ is a phase induced by the matter effects, which is not important for our purposes. See Ref. [5,6] for notation. Because P_1 depends on $\Delta m^2 \cos 2\theta$, the half of the parameter space traditionally considered $0 \le \theta \le \frac{\pi}{4}$ (the light side) is physically inequivalent to the other half $\frac{\pi}{4} < \theta \le \frac{\pi}{2}$ (the dark side). However, all data analysis have been reported on the $(\Delta m^2, \sin^2 2\theta)$ plane with positive Δm^2 only for solar neutrino experiments and hence only half of the parameter space has been analyzed. Even though the dark side has been studied in the context of three-flavor [7] and four-flavor [8] neutrino oscillations, the importance of studying both halves for the simplest case of two-flavor oscillations has been largely ignored in the literature. There also appeared to be a misconception in the literature that physics was discontinuous at maximal mixing $\theta = \frac{\pi}{4}$. For instance, matter effects in the Earth were once thought to disappear as the mixing approached maximal. However, the authors of Ref. [9] emphasized that the matter effects remain important even for the maximal mixing, and the present authors further showed that physics is completely continuous beyond $\theta = \frac{\pi}{4}$ [6]. One can still retain the dark side with only $0 \le \theta \le \frac{\pi}{4}$ if a separate parameter space with $\Delta m^2 < 0$ is added. This is indeed what Super-Kamiokande did in the case of $\nu_\mu \leftrightarrow \nu_s$ oscillations of atmospheric neutrinos [10]. However, as argued in [6], it is more natural to use $0 \le \theta \le \frac{\pi}{2}$ with the fixed sign of Δm^2 to exhibit the continuity of the physics between the two halves of the parameter space.

Part of the reason why the dark side has been neglected in the literature is that it is impossible to obtain ν_e survival probabilities less than one half when the two mass eigenstates are incoherent, *i.e.*, when the last term in Eq. (3) is absent. (This occurs in the so-called "MSW region" $10^{-8} \lesssim \Delta m^2 \lesssim 10^{-3}$ eV^2 [11].) Indeed, the data from the Homestake experiment [3] used to be about a quarter of the SSM prediction, and this could have been an argument for dropping the dark side entirely in the MSW region. However, the change from BP95 [12] to BP98 [13] calculations increased the Homestake result to about a third of the SSM with a relatively large theoretical uncertainty. Therefore it is quite possible that the "MSW solutions" extend to the dark side as well. Moreover, some people question the SSM and/or the Homestake experiment, and perform fits by ignoring either (or both) of them [14]. We show below that some of the MSW solutions indeed extend to the dark side and hence it is necessary to explore the dark side experimentally. If we further relax the theoretical prediction on the ^{8}B solar neutrino flux and/or ignore one of the solar neutrino experiments in the global fit, the preferred regions extend even deeper into the dark side.

Another possible reason for disregarding the dark side is that the so-called "vacuum oscillation region" ($\Delta m^2 \lesssim 10^{-9}$ eV^2) was believed to be the same in the light and dark sides. This is because P_1 approaches $\cos^2\theta$ for $\Delta m^2 \ll 10^{-9}$ eV$^2(E/\mathrm{MeV})$ and Eq. (3) reduces to Eq. (2). It is remarkable, however, that low-energy (especially *pp*) neutrinos do not reach this limit for $\Delta m^2 \gtrsim 10^{-10}$ eV^2 and hence the preferred regions are different in the light and the dark sides [15]. This observation also implies that the separation of the MSW region and the vacuum oscillation region as traditionally done in the global fits is artificial and misleading. It is important to study the entire range of Δm^2 continuously.

If $\sin^2 2\theta$ is not a good parameter, what is the alternative? Two suggestions have been made in the literature. One is $\sin^2\theta$, which is natural since the matter effect depends directly on $\sin^2\theta$ [6]. If plotted on the linear scale, pure vacuum oscillations would yield physics reflection-symmetric around $\sin^2\theta = 0.5$. If plotted on the log scale, the reflection symmetry is lost, but it is still a useful parameterization as physics is completely continuous and smooth from the light to the dark side. Another possible parameterization is $\tan^2\theta$, which retains the reflection symmetry for pure vacuum oscillation around $\tan^2\theta = 1$ if plotted on the log scale [7,6]. We employ $\tan^2\theta$ for the analysis below because we would like to use the log scale to present the MSW solutions as well as the importance of the matter effect on the "vacuum oscillation" region at the same time. Note that the Jacobian from $\sin^2\theta$ or $\tan^2\theta$ to $\sin^2 2\theta$ is singular at $\theta = \frac{\pi}{4}$ and plots with $\sin^2 2\theta$ will display unphysical singular behavior there [6].

We next present the results of global fits to the current solar neutrino data from water Cherenkov detectors (Kamiokande and Super-Kamiokande) [16], a chlorine target (Homestake) [3] and gallium targets (GALLEX and SAGE) [17] on the full parameter space. We do not include the spectral data from Super-Kamiokande [18] as it appears to be still evolving with time. The fit is to the event rates measured at these experiments only. In computing the rates we include not only the *pp*, ^{7}Be, and ^{8}B neutrinos, but also the ^{13}N, ^{15}O, and *pep* neutrinos. We use Eq. (3) with P_c computed in the exponential

approximation for the electron number density profile in the Sun, and properly account for neutrino interactions in the Earth during the night with a realistic Earth electron number density profile by numerically solving Schrödinger equation as described in [6]. Since the mixing angle at the production point in the Sun's core depends on the electron number density, we integrate over the production region numerically. We treat the correlations between the theoretical uncertainties at different experiments following Ref. [7]. To insure a smooth transition between the MSW and the vacuum oscillation region, we integrate over the energy spectrum (including the thermal broadening of the ^{7}Be neutrino "line") for $\Delta m^2 \leq 10^{-8}$ eV^2 and average the neutrino fluxes over the seasons. For $\Delta m^2 > 10^{-8}$ eV^2 we treat the two mass eigenstates as incoherent. Results are completely smooth at $\Delta m^2 = 10^{-8}$ eV^2, as expected. This allows us to fit the data from $\Delta m^2 = 10^{-11}$–10^{-3} eV^2 all at once, unlike previous analyses which separate out the "vacuum oscillation region" from the rest.

As was mentioned earlier, we take the global fit to the currently available data only as indicative of the ultimate result because we expect much better data to be collected in the near future to eventually supersede the current data set. We would like to keep our minds open to surprises such as the possibility that one of the earlier experiments was not entirely correct or that the theoretical uncertainty in the flux prediction was underestimated. In this spirit, we employ more conservative attitudes in the global fit than most of the analyses in the literature in the following three possible ways. (1) We allow higher confidence levels, such as 3 σ. (2) We relax the theoretical prediction on the neutrino flux. (3) We ignore some of the experimental data in the fit.

The global fit results are presented in Fig. 1 at the 2 σ (95% CL) and 3 σ (99.7% CL) levels defined by $\chi^2 - \chi^2_{min}$ for two degrees of freedom. It is noteworthy that both the LMA and LOW solutions (we use the nomenclature introduced in [19]) extend to the dark side at the 3 σ level. At 99% CL, however, the LMA solution is confined to the light side. This result is consistent with the two-flavor limit of the three-flavor analysis in [7] and the four-flavor analysis in [8], where the spectral data is included and the LOW solution extends into the dark side at 99% CL. Another interesting fact is that the LOW solution is smoothly connected to the VAC solution, where the preferred region is clearly asymmetric between the light and the dark sides. Note that, at $\Delta m^2 \sim 10^{-9}$ eV^2, the allowed region is bigger in the dark side. The region $10^{-9} < \Delta m^2 < 10^{-8}$ eV^2 was, to the best of the authors' knowledge, never studied fully in the literature and this result demonstrates the need to study the entire Δm^2 region continuously without the artificial separation of the "MSW region" and "vacuum oscillation region," as traditionally done in the literature.

We next present a fit where the theoretical prediction

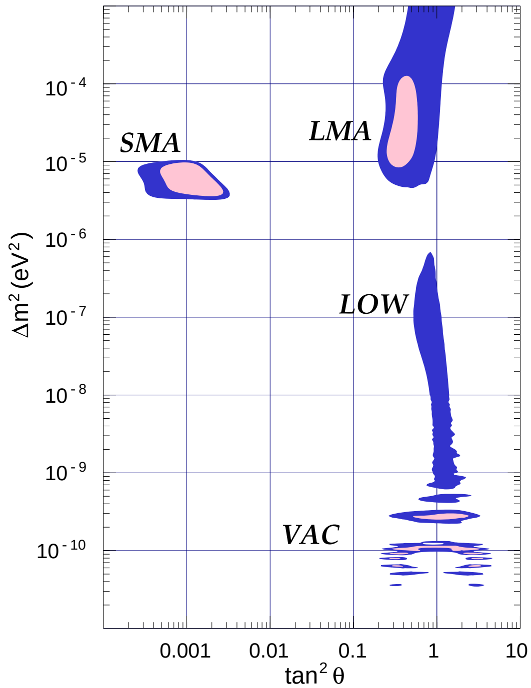

FIG. 1. A global fit to the solar neutrino event rates at chlorine, gallium and water Cherenkov experiments. The regions are shown at 2 σ (light shade) and 3 σ (dark shade) levels. The region $\tan^2\theta > 1$ is the dark side $\theta > \frac{\pi}{4}$.

of the ^{8}B flux is relaxed. Even though the helioseismology data constraints the sound speed down to about 5% of the solar radius [13], the core region where ^{8}B neutrinos are produced is still not constrained directly. Given the sensitive dependence of the ^{8}B flux calculation on the core temperature $\Phi_{^8B} \sim T^{22}$, we may consider it as a free parameter in the fit. This can be done within the formalism of Ref. [7] by formally sending the error in $C_{\rm Be}$ to infinity. The result is presented in Fig. 2. The preferred region extends more into the dark side than the previous fit. Even though the LMA and LOW solutions are connected in this plot, the lack of a large day-night asymmetry at Super-Kamiokande would eliminate the range $3 \times 10^{-7} \lesssim \Delta m^2 \lesssim 10^{-5}$ eV^2 for $0.2 \lesssim \tan^2\theta < 1$ [16]. It is important for Super-Kamiokande to report their exclusion region on the dark side.

Finally, we present a fit where the event rate measured at the Homestake experiment is not used in Fig. 3. This may be a sensible exercise given that the neutrino capture efficiency was never calibrated in this experiment. The preferred region extends into the dark side even at the 95% CL. Note also the asymmetry between the dark and the light sides even for $\Delta m^2 < 10^{-9}$ eV^2.

We expect the data of the current and next generation of solar neutrino experiments, such as Super-Kamiokande, SNO, GNO, Borexino, KamLAND, to eventually supersede the current data set. Therefore we regard the above global fits only as estimates of the ultimate results. The most important point is that all exper-

imental collaborations should report their results, both exclusion and measurements, on both sides of the parameter space, without unnecessary theoretical bias towards the light side. We strongly urge the experimental collaborations to consider this point.

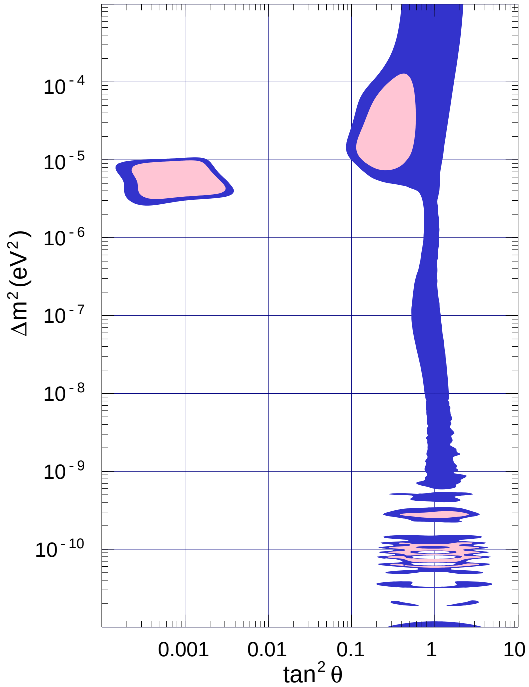

FIG. 2. A global fit to the solar neutrino event rates at chlorine, gallium and water Cherenkov experiments, where the ^{8}B flux is treated as a free parameter. Contours are shown at 2 σ (light shade) and 3 σ (dark shade).

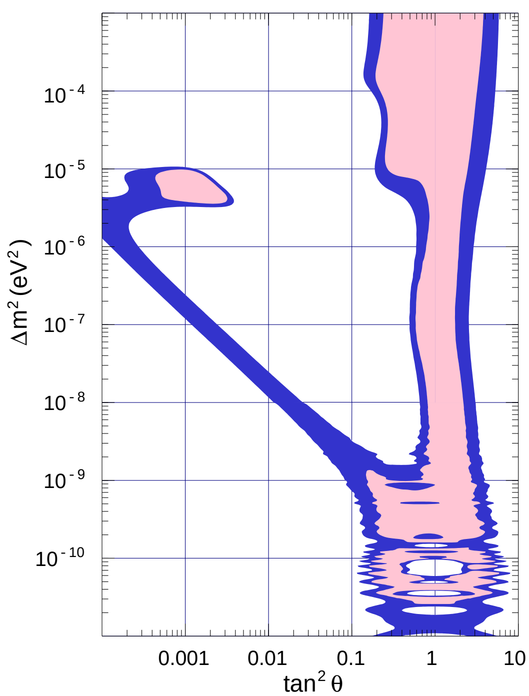

FIG. 3. A global fit to the solar neutrino event rates at the gallium and water Cherenkov experiments but not at the chlorine experiment. Contours are shown at 2 σ (light shade) and 3 σ (dark shade).

---

* This work was supported in part by the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY-95-14797.

[1] Super-Kamiokande Collaboration (Y. Fukuda *et al.*), *Phys. Rev. Lett.* **81**, 1562 (1998), hep-ex/9807003.

[2] Solar Neutrinos: The First Thirty Years, *eds.*, J.N. Bahcall, R. Davis, P. Parker, A. Smirnov, R. Ulrich, Reading, USA, Addison-Wesley, 1995, Frontiers in Physics. Vol 92.

[3] B.T. Cleveland *et al.*, *Astrophys. J.* **496**, 505 (1998).

[4] L. Wolfenstein, *Phys. Rev.* **D 17**, 2369 (1978); S.P. Mikheyev and A.Yu. Smirnov, *Yad. Fiz. (Sov. J. of Nucl. Phys.)* **42**, 1441 (1985).

[5] A. de Gouvêa, A. Friedland, and H. Murayama, *Phys. Rev.* **D60**, 093011 (1999), hep-ph/9904399.

[6] A. de Gouvêa, A. Friedland, and H. Murayama, hep-ph/9910286.

[7] G.L. Fogli *et al.*, hep-ph/9912231.

[8] C. Giunti, M.C. Gonzalez-Garcia, and C. Peña-Garay, hep-ph/0001101.

[9] A.H. Guth, L. Randall and M. Serna, *JHEP* **9908**, 018 (1999), hep-ph/9903464.

[10] T. Kajita, talk presented at PASCOS99, Granlibakken, Lake Tahoe, California, December 10-16, 1999.

[11] For instance, integration over neutrino energies washes out the oscillatory factor. In the case of ^{7}Be neutrinos, this occurs for $\Delta m^2 \gtrsim 10^{-8}$ eV^2 due to the thermal broadening effect. For neutrinos with continuous spectra, the washout occurs at even lower Δm^2. See [5,15] for more details.

[12] J.N. Bahcall and M.H. Pinsonneault, *Rev. Mod. Phys.* **67**, 781 (1995), hep-ph/9505425.

[13] J.N. Bahcall, S. Basu, and M.H. Pinsonneault, *Phys. Lett.* **B433**, 1 (1998), astro-ph/9805135.

[14] For a recent study along this line, see R. Barbieri *et al.*, *JHEP* **9812**, 017 (1998), hep-ph/9807235.

[15] A. Friedland, hep-ph/0002063.

[16] Super-Kamiokande Collaboration (Y. Fukuda *et al.*), *Phys. Rev. Lett.* **82**, 1810 (1999), hep-ex/9812009.

[17] GALLEX Collaboration (W. Hampel *et al.*), *Phys. Lett.* **B447**, 127 (1999); SAGE Collaboration (J.N. Abdurashitov *et al.*), *Phys. Rev.* **C60**, 055801 (1999).

[18] Super-Kamiokande Collaboration (Y. Fukuda *et al.*), *Phys. Rev. Lett.* **82**, 2430 (1999), hep-ex/9812011.

[19] J.N. Bahcall, P.I. Krastev, and A.Yu. Smirnov, *Phys. Rev.* **D 58**, 096016 (1998), hep-ph/9807216.